Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*6859654903*

**MATHEMATICS** **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2022**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.




**[Turn over**

**1** Solve the equation $3x + 2 = \dfrac{2}{x - 1}$. [3]

**2** The equation of a curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 12\left(\frac{1}{2}x - 1\right)^{-4}$. It is given that the curve passes through the point $P(6, 4)$.

**(a)** Find the equation of the tangent to the curve at $P$. [2]

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**(b)** Find the equation of the curve. [4]

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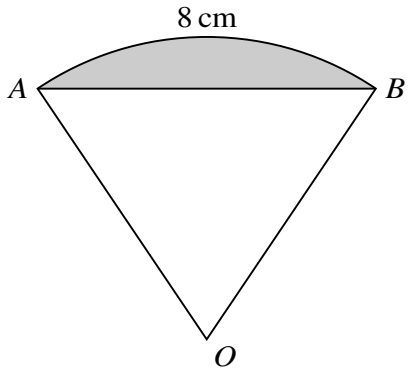
**3** A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and $a$ is a constant. The curve has a stationary point at the point $P$, which has $x$-coordinate 9.

Find the $y$-coordinate of $P$. [5]

**4** The coefficient of $x^2$ in the expansion of $\left(1 + \frac{2}{p}x\right)^5 + (1 + px)^6$ is 70.

Find the possible values of the constant $p$. [6]

**5**




The diagram shows a sector $OAB$ of a circle with centre $O$. The length of the arc $AB$ is 8 cm. It is given that the perimeter of the sector is 20 cm.

**(a)** Find the perimeter of the shaded segment. [4]

**(b)** Find the area of the shaded segment. [2]

**6** **(a)** Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where $a$, $b$ and $c$ are constants to be found. [3]

**(b)** Hence solve the equation $\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

**7** A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

**(a)** Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

**(b)** Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

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**(c)** Find the greatest total depth in the ground which could theoretically be achieved. [2]

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**8** The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where $p$ is a constant.

**(a)** Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

**(b)** Express $\mathrm{f}^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx-d}$, where $a$, $b$, $c$ and $d$ are integers. [4]

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**(c)** Hence state the value of $p$ for which $\mathrm{f}^{-1}(x) \equiv \mathrm{f}(x)$. [1]

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**9** Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$\mathrm{f}(x) = x^2 - 4x + 9,$$
$$\mathrm{g}(x) = 2x^2 + 4x + 12.$$

**(a)** Express $\mathrm{f}(x)$ in the form $(x - a)^2 + b$. [1]

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**(b)** Express $\mathrm{g}(x)$ in the form $2[(x + c)^2 + d]$. [2]

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**(c)** Express $g(x)$ in the form $kf(x + h)$, where $k$ and $h$ are integers. [1]

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**(d)** Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

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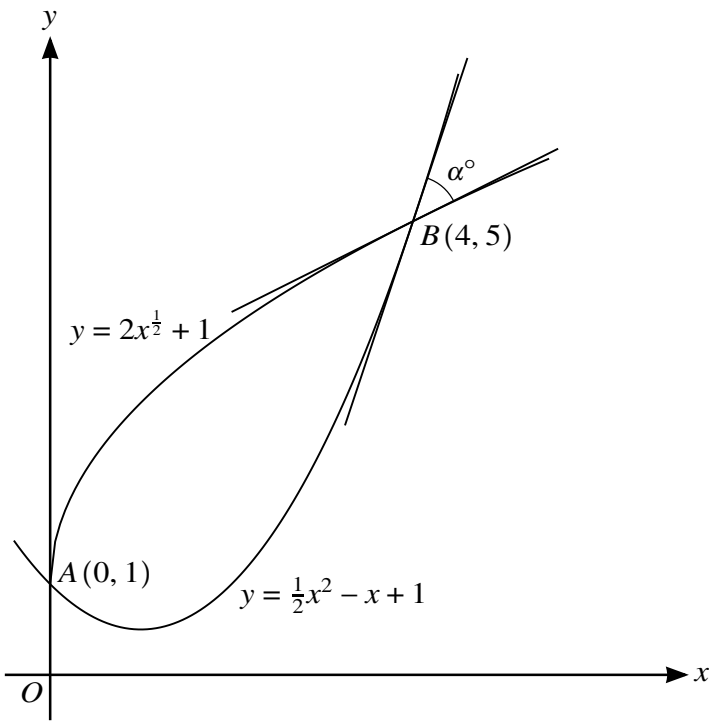
**10**




Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0,\ 1)$ and $B\,(4,\ 5)$, as shown in the diagram.

**(a)** Find the area of the region between the two curves. [5]

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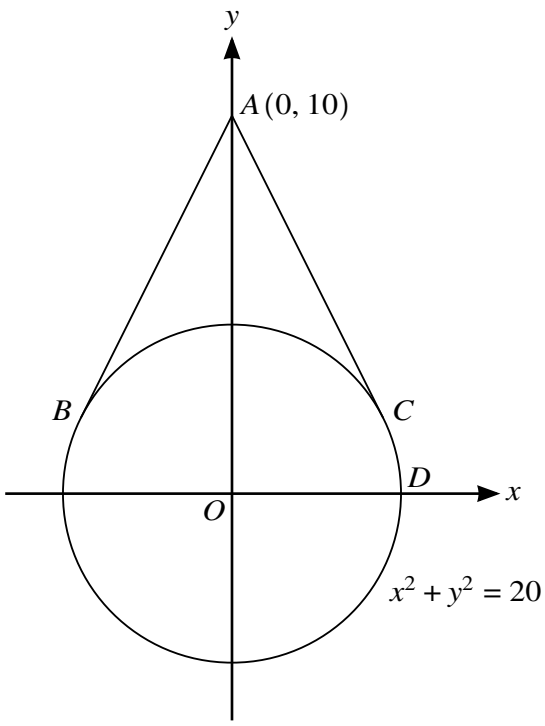
The acute angle between the two tangents at $B$ is denoted by $\alpha°$, and the scales on the axes are the same.

**(b)** Find $\alpha$. [5]

**11**




The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points $B$ and $C$ pass through the point $A$ (0, 10).

**(a)** By letting the equation of a tangent be $y = mx + 10$, find the two possible values of $m$. [4]

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**(b)** Find the coordinates of $B$ and $C$. [3]

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The point $D$ is where the circle crosses the positive $x$-axis.

**(c)** Find angle $BDC$ in degrees. [3]

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## Additional Page

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